ANALYSIS OF CHAOTIC BEHAVIOUR IN LUMPED-DISTRIBUTED CIRCUITS APPLIED TO PRACTICAL MICROWAVE OSCILLATORS

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ABSTRACT : A general method for the analysis of pre-chaotic and chaotic behaviours in lumped-distributed circuits has been developed. Examples of practical microwave oscillators, namely GUNN and IMPATT oscillators were designed. The simulation and measurement results showed that these oscillators possess regions of periodic, quasi-periodic, and chaotic spectra. The method could be used by circuit designers to design chaos free oscillators.

I. INTRODUCTION

A significant number of microwave lumped-distributed nonlinear networks possess highly nonlinear elements. These networks are also of very high order because of the large number of elements required to simulate devices and circuits at microwave frequencies. These two properties make microwave networks very prone to chaotic behaviour. Chaotic behaviour is more likely to take place in nonlinear microwave circuits containing negative resistance diodes as active elements and coaxial transmission lines, strip lines, cavity resonators ,or tuning varactors as passive elements. In circuits containing waveguides, the chaotic behaviour is less probable due to the waveguide low frequency cut off property. An analysis procedure for lumped-distributed circuits has been developed to study the chaotic phenomena of microwave circuits [1]

Two examples of microwave oscillators containing single and multi-nonlinear elements (the GUNN and IMPATT diodes) were designed by using the proposed procedure. Different bifurcation parameters were considered, namely the slope of the nonlinear function, the delay of a transmission line, and the bias current of the oscillator circuit. Different routes to chaos were displayed, namely quasiperiodicity, intermittency, noisy periodicity, and period doubling.

II. ANALYSIS PROCEDURE

The state and output equations describing a general nonlinear lumped-distributed network [2] are given by,

$$\begin{bmatrix} \dot{x}_1(t) \\ x_2(t+T_i) \end{bmatrix} = \begin{bmatrix} A_1 & A_2 \\ A_3 & A_4 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} + Bu(t) + B_n F_n(x_1, x_2, u) \qquad (1)$$

$$y(t) = \begin{bmatrix} C_1 & C_2 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} + Du(t) + D_n F_n(x_1, x_2, u) \qquad (2)$$

where x_1, x_2 are the lumped and distributed state vectors,
F_n is the vector of nonlinear functions,
u,y are the input and output vectors ,
T_i is the delay of the ith transmission line,
A, B, C, D are real matrices.

The conditions at the equilibrium points (D.C. solutions) are given by,

$$\dot{x}_1(t) = 0 \quad , \quad x_2(t+T_i) = x_2(t) \qquad (3)$$

Applying conditions (3) into (1), the following matrix equation for the equilibria can be obtained,

$$\begin{bmatrix} A_1 & A_2 \\ A_3 & A_4 - I_m \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = -B\ u - B_n\ F_n(x_1, x_2, u) \qquad (4)$$

where I_m is a unit matrix of order m.

The solution of (4) gives the coordinates of l equilibrium points.

$$x_{Qi} = \begin{bmatrix} x_{1Q_i} & x_{2Q_i} \end{bmatrix}^T \qquad i.1,2,...,l \qquad (5)$$

where x_{1Qi}, x_{2Qi} are the coordinates of the lumped and distributed state vectors at the ith equilibrium point.

TH 4B

For circuits containing a single nonlinear element, the system stability criteria of the linear system and the graph of the nonlinear element can be combined in the i-v plane of the nonlinearity. Thus the existence of equilibria, their positions, stability can be obtained. In this case one can identify two types of oscillations, $(-\alpha,\beta)$ and multilevel oscillations [3]-[4]. The stability study is also done locally at each equilibrium point for circuits containing more than one nonlinear element. Movement of the equilibria, their stability properties as a function of a bifurcation parameter, birth of new equilibria, and points of changing the local stability are required information to judge the structural stability of the system.

III. GUNN DIODE OSCILLATOR

The Gunn diode oscillator shown in Fig.(1) is an example of a lumped-distributed circuit containing one nonlinear element.

1. Stability Criteria of the GUNN Oscillator

With the choice of the relative delay, $T=T_1/T_3$, to be the bifurcation parameter, the stability of the linearized system can be constructed in the slope-delay plane and in the (i-v) plane of the diode characteristics. Applying the multilevel oscillation theorem [3]-[4] the stability criteria of two qualitatively different regions can be identified :

i. Stable Region independent of delay : This region is given by, $m < m_{lmin}$, $m > m_{lmin}$ as indicated in Fig(2.a).

ii. Region of Possible oscillating Solutions: This region is given by, ($m_{lmin} < m < m_{lmin}$), ($T_a < T < T_b$) ,where T_a, T_b are the minimum and maximum relative delay values for a particular value of $m=m_o$, (see Fig.(2.a)).

The absolute stability sector in the (i-v) plane of the nonlinear function corresponding to the region between the points T_a and T_b in the slope-delay plane is shown in Fig.(2.b) for a constant value of the relative delay T_1/T_3. For a possible oscillatory solution, m_o must lie in the range, $m_u < m_o < m_l$. It is clear from Fig.(2.a) that the boundaries m_u and m_l are moving as the relative delay T_1/T_3 varies. Oscillatory solutions were found with $m= m_o$, R_l =-0.569, T_1=26.36 ps, T_2=53.2 ps, and T_3 varied as a bifurcation parameter. Within this oscillatory region three regions were found namely the undamped oscillation region, the noisy periodic region, and the chaotic region. The properties of the eigenvalues within each of these regions were studied to determine whether these regions can be predicted from the eigenvalues. The equilibrium point at the origin is unstable, while the other symmetric non-zero equilibria have different stability properties in each of the following sub-regions :

- In the undamped oscillation region, the eigenvalues of the non-zero equilibria are stable with relatively large negative real parts compared to the stable periodic region.
- In the noisy periodic region, the non-zero equilibria are non-hyperbolic (two eigenvalues have zero real parts).
- In the region assigned chaotic, the non-zero equilibria are non-stable (some eigenvalues have negative real parts and some have positive real parts).
- In the region of stable periodic solutions (periodic solutions with countable periods), the non-zero equilibria are stable with some of the eigenvalues having very small negative real parts.

2. Simulation Results of the Gunn oscillator

The circuit was simulated using our time domain program [5] in each of the above mentioned regions by varying the value of the delay of the tuning transmission line, T_3. For T1=26.36 ps, T_2=53.2 ps, and T_3=18.8 ps, a harmonic solution in the undamped oscillation region is obtained. In this region we have harmonic oscillations, the amplitude and frequency of which are controlled by the delay T_3. The simulations revealed that a reverse bifurcation of bands (noisy periodic) transition associated with period three and period doubling takes place. In the other region assigned chaotic , the intermittent period four solution for which the bursts distribution becomes random at T_3=15.31 ps was observed. At T_3=14.4 ps a harmonic solution with the fundamental frequency was again observed. It is important to note that the harmonic solution obtained in the stable periodic region is less in amplitude than the one obtained in the undamped region. This clarifies the importance of the specified regions of performance in Fig.(2.a) to the designers.

3. Practical Results of the Gunn Oscillator

The measured spectrum of the oscillator indicated the basic design frequency f_o and a period doubling state for which the two frequency components appear at f_o and $f_1=f_o/2$. The increase of the bifurcation parameter (the delay T_3) resulted in new frequency components incommensurate with the basic frequency. Further increase of the bifurcation parameter led to the breaking of the quasi-periodicity and mode locking followed by spectral broadening.The quasiperiodic and chaotic spectra are shown in Fig.(3).

IV. IMPATT DIODE OSCILLATOR

The IMPATT diode model proposed by Ganett and Chua

[6] was implemented in the analysis procedure. The complete circuit for the oscillator is shown in Fig.(4).

1. Simulation Results of IMPATT oscillator

The circuit was simulated using a time domain simulator [5] in which the nonlinear elements are replaced by linear elements and nonlinear controlled sources. The simulation of the circuit resulted in the following modes of behaviour:

i. Normal Operation Mode: For the value of the bias current I_o=63 mA,and the delay ratio $0.62<T_3/T_3<0.65$, the simulation showed a harmonic solution with the fundamental frequency, f_o=5.18 GHz, which is in good agreement with the design values, f_o=5 Ghz.

ii. Period Doubling Operation Mode: Keeping the value of the bias current at I_o=63 mA, and increasing the delay ratio, period doubling followed by period four solutions are observed for delay ratios of 0.655 and 0.662, respectively.

iii.Quasiperiodic and Chaotic Operation Modes: In this case the bias current was increased to 70 mA, and the delay ratio to be 0.662, a quasi-periodic spectrum was obtained. The spectrum is composed of two independent components f_o and f_1 and some of their integer combinations . Further increase of the delay ratio resulted in a chaotic solution.

3. Practical Results of IMPATT diode oscillator

The spectrum of the oscillator in the normal mode is shown in Fig.(5.a). The increase of the tuning length resulted in a small shift in the frequency of the fundamental and three subharmonics $f_o/4$, $f_o/2$, $3f_o/4$ which is a period four solution. Increasing the value of the bias current to 70 mA resulted in a quasi-periodic spectrum with two independent frequencies. Further increase in the relative delay resulted in a broad band noise-like spectrum as shown in Fig.(5.b). Agreements between simulations and practice have been achieved.

VI. CONCLUSIONS

The study of the chaotic behaviour of two widely used lumped-distributed microwave oscillators, namely the GUNN and IMPATT oscillators, has shown that many effects, previously attributed to noise, parametric instability, or power supply oscillations are in fact controllable and can be avoided. Regions of chaos free oscillations, noisy periodic oscillations, quasi-periodic oscillations, and chaotic oscillations are predicted and verified by measurements. It is of major importance to find regions of different modes of behaviour of such devices to help the designer.

References

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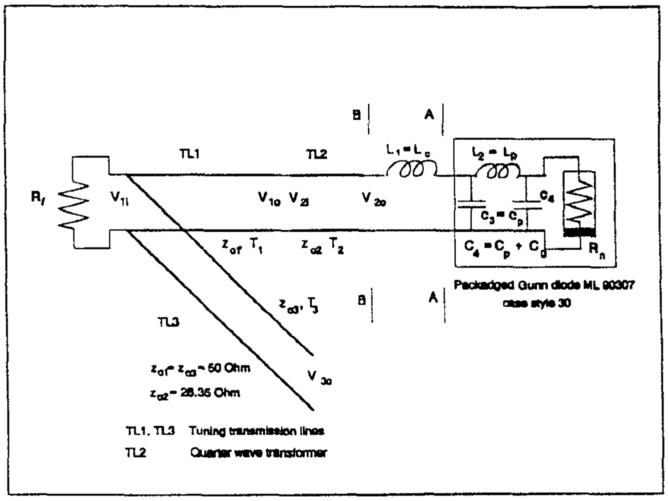


Fig.(1) The GUNN diode oscillator circuit.

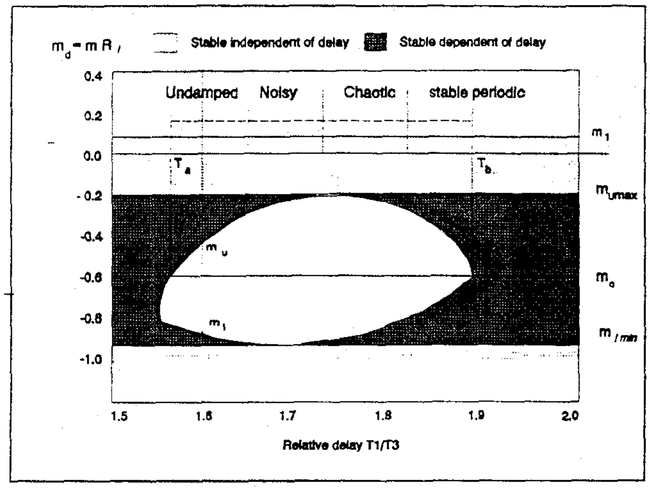


Fig.(2.a) Absolute stability criteria of the GUNN oscillator in the slope-delay plane T_1=26.36 ps, T_2=53.2 ps.

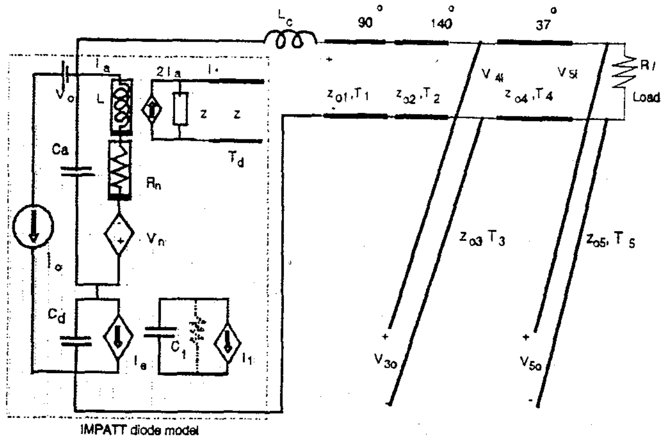


Fig.(4) The IMPATT diode oscillator circuit.

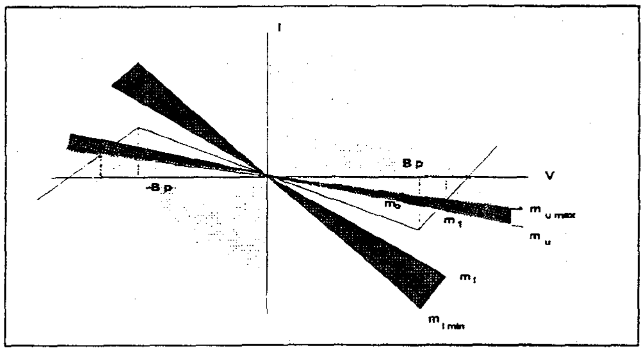
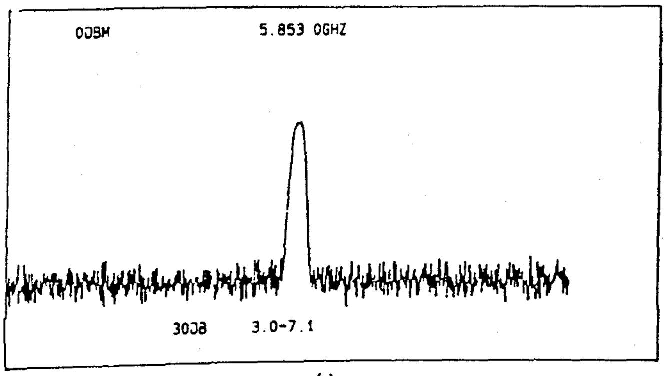
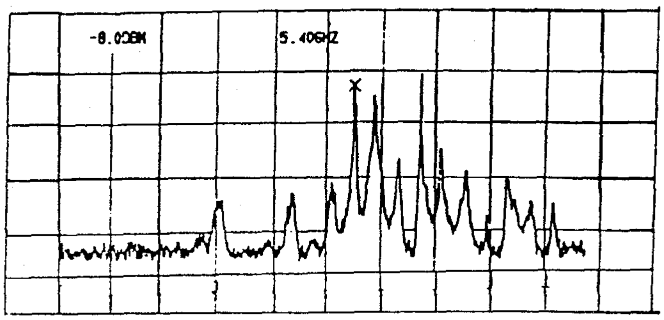


Fig.(2.b) Absolute stability criteria in the (i-v) plane of the nonlinear characteristics of the GUNN diode.



(a)



(b)

Fig.(5) Measured spectrum of the IMPATT oscillator
(a) Harmonic (b) Chaotic.

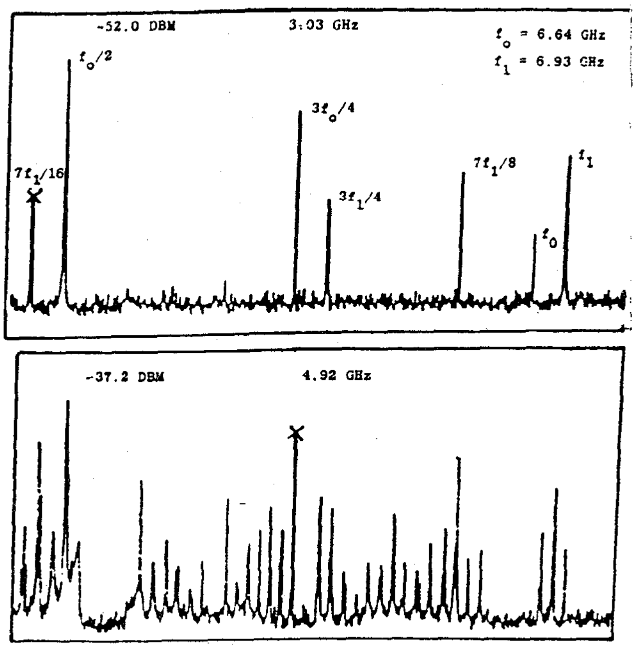


Fig.(3) Measured spectrum of the GUNN diode in the nonperiodic mode (a) quasiperoidic (b) Chaotic.